

## Conferință Lunară

*Classical  $p$ -Banach spaces,  $0 < p < 1$*

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**Miercuri 20 septembrie 2017, ora 13:00  
IMAR, Amfiteatrul Miron Nicolescu, parter**

**Abstract:** This a survey concerning results obtained in the early '80s by the author [14], [15], [16], [17] about some classical  $p$ -Banach spaces,  $0 < p < 1$ .

Also some answers to the problems considered in these papers are given.

A vector space  $E$  on the complex field is a  $p$ -Banach space, where  $0 < p < 1$ , if there is a function  $\|\cdot\| : E \rightarrow \mathbb{R}_+$ , such that:

1.  $\|x\| = 0$  if and only if  $x = 0$ .
2.  $\|\alpha x\| = |\alpha| \cdot \|x\| \quad \forall x \in E, \alpha \in K$ .

$$3. \quad \|x + y\|^p \leq \|x\|^p + \|y\|^p \quad \forall x, y \in E,$$

and the space  $E$  is complete with respect to the topology generated by the  $\|\cdot\|$ .

A table of contents of my talk:

1. Examples of  $p$ -Banach spaces
2. Why study  $p$ -Banach spaces?
3. Some topics in the theory of  $p$ -Banach spaces
4. Complemented subspaces
5. Positive complemented sublattices
6. Unicity of the unconditional and symmetric bases
7. Banach envelope
8.  $p$ -convex Banach lattices,  $0 < p < 1$

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